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INVENTORY SYSTEM WITH SHORTAGES AND WEIBULL DETERIORATION FUNCTION

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ABSTRACT

Factors such as demand, deteriorating rate, and so on should be taken into consideration in the deteriorating inventory study. Among them, demand acts as driving force of the entire inventory system and the deteriorating rate stands for the characteristics of the deteriorating items. Other factors like price discount, allow shortage or not, inflation, and the time-value of money are also important in the study of deteriorating items inventory.

KEY WORDS : Inventory, deteriorating rate

INTRODUCTION

Inventory cost is an important part of the enterprise operation cost. For deteriorating items, especially those with high deteriorating rate, deterioration is a key characteristic and its impact on modeling of inventory systems cannot be neglected. So the deterioration rate should be taken into consideration in the development of inventory strategy. For different kinds of enterprises, the emphasis on the deteriorating items inventory study is different. For the seller of deteriorating items, the current studies can be divided into two types; the first type emphasizes the inventory strategies for the retailer of the deteriorating items, the second type focuses on the inventory policy under a two-warehouse system. For the manufactures of deteriorating items, the current emphasis is on developing an optimal production-inventory strategy. So, this paper has divided the present studies on deteriorating items inventory for a single enterprise into three categories as stated above. In this paper an attempt has been made to develop an inventory model for infinite planning horizon with exponentially increasing demand rate. It can be noticed that deterioration does not depends upon time only. It can affect due to whether conditions, humidity, storage conditions etc. therefore it is more realistic to consider deterioration rate as two parameter weibul distribution function. Shortage are allowed and fully backlogged. The holding cost considered a linear function of time. The optimal solution of t he proposed inventory model is derived and considered same cases.

ASSUMPTIONS & NOTATIONS

- The replenishment size is constant and production is instantaneous during prescribed time period T of each cycle.
- Lead time is zero.
- Shortage are permitted and completely accumulated
- Demand rate $D(t)=\frac{d}{(e-1)T}e^{t/T}$ at any time t.
- Deterioration rate $\theta=\alpha\beta t^{\beta-1},\ 0<\alpha<1,\ \beta\geq 1$
- Holding cost $C_1=h+\gamma t$ per unit.
- $C_1,\ C_2$ are the cost of each item, shortage cost per unit per unit time respectively.
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ANALYSIS FOR THE SYSTEM:

Let I(t) be the current stock level at any time t.

$$\frac{dI(t)}{dt}+\theta I(t)=-\frac{d}{(e-1)T}e^{\frac{t}{T}}, \qquad 0\le t\le t_1 \qquad \ldots(1.1)$$

$$\frac{dI(t)}{dt}=-\frac{d}{(e-1)T}e^{\frac{t}{T}}, \qquad t_1\le t\le T \qquad \ldots(1.2)$$

By equation (1.1), we have

$$\frac{dI(t)}{dt}+\alpha\beta t^{\beta-1}\cdot I(t)=-\frac{d}{(e-1)}e^{\frac{t}{T}}$$

$$\text{I.F.}=e^{\int\alpha\beta t^{\beta-1}dt}=e^{\alpha t^{\beta}}.$$

Solution of equation (1.1) is given by

$$I(t)e^{\alpha t^{\beta}}=-\int\frac{d}{(e-1)T}e^{\frac{t}{T}}e^{\alpha t^{\beta}}dt+B$$

where B is the constant of integration.

$$=-\frac{d}{(e-1)T}\int e^{\frac{t}{T}}\left(1+\alpha t^{\beta}\right)dt+B.$$

After expanding $e^{\frac{t}{T}}$ by Taylor's series and neglecting higher order terms of $\frac{t}{T}$ greater than 1 $\left(\because\frac{t}{T}<1\right)$, we get

$$I(t)e^{\alpha t^{\beta}}=-\frac{d}{(e-1)T}\int\left(1+\frac{t}{T}\right)\left(1+\alpha t^{\beta}\right)dt+B$$

$$=-\frac{d}{(e-1)T}\int\left(1+\frac{t}{T}+\alpha t^{\beta}+\frac{\alpha}{T}t^{\beta+1}\right)dt+B$$

$$=-\frac{d}{(e-1)T}\left[t+\frac{t^2}{2T}+\frac{\alpha t^{\beta+1}}{\beta+1}+\frac{\alpha t^{\beta+2}}{T(\beta+2)}\right]+B. \qquad \ldots(1.3)$$

At $t=0$, $I(t)=S$, then from equation (1.3), we have

$$I(t)e^{\alpha t^{\beta}}=-\frac{d}{(e-1)T}\left[t+\frac{t^2}{2T}+\frac{\alpha t^{\beta+1}}{\beta+1}+\frac{\alpha t^{\beta+2}}{T(\beta+2)}\right]+S. \qquad \ldots(1.4)$$

At $t=t_1$, $I(t)=0$, then from equation (84), we have

$$S=\frac{d}{(e-1)T}\left[t_1+\frac{t_1^2}{2T}+\frac{\alpha t_1^{\beta+1}}{\beta+1}+\frac{\alpha t_1^{\beta+2}}{T(\beta+2)}\right]. \qquad \ldots(1.5)$$

Substituting the value of S in equation (1.4) from equation (1.5), then

$$I(t)=\frac{d}{(e-1)T}\left[t_1+\frac{t_1^2}{2T}+\frac{\alpha t_1^{\beta+1}}{\beta+1}+\frac{\alpha t_1^{\beta+2}}{T(\beta+2)}-t\right.$$

$$-\frac{t^2}{2T}-\frac{\alpha t^{\beta+1}}{\beta+1}-\frac{\alpha t^{\beta+2}}{T(\beta+2)}\Bigg](1-\alpha t^{\beta}). \qquad \ldots(1.6)$$

Neglecting higher order terms of α, we get from equation (1.6)

$$I(t)=\frac{d}{(e-1)T}\Bigg[t_1-t+\frac{t_1^2}{2T}-\frac{t^2}{2T}-\alpha t_1 t^{\beta}+\alpha t^{\beta+1}-\frac{\alpha t_1^2 t^{\beta}}{2T}$$

$$+\frac{\alpha t^{\beta+2}}{2T}+\frac{\alpha t_1^{\beta+1}}{\beta+1}+\frac{\alpha t_1^{\beta+2}}{T(\beta+2)}-\frac{\alpha t^{\beta+1}}{\beta+1}-\frac{\alpha t^{\beta+2}}{T(\beta+2)}\Bigg]$$

$$I(t)=\frac{d}{(\ell-1)T}\Bigg[t_1+\frac{t_1^2}{2T}+\frac{\alpha t_1^{\beta+1}}{\beta+1}+\frac{\alpha t_1^{\beta+2}}{T(\beta+2)}-t$$

$$-\frac{t^2}{2T}-\alpha t_1 t^{\beta}-\frac{\alpha t_1^2 t^{\beta}}{2T}+\frac{\alpha\beta t^{\beta+1}}{\beta+1}+\frac{\alpha\beta t^{\beta+2}}{2T(\beta+2)}\Bigg] \ldots(1.7)$$

Solution of equation (1.2) is given by

$$I(t)=\frac{d}{e-1}\left(e^{\frac{t_1}{T}}-e^{\frac{t}{T}}\right). \qquad \ldots(1.8)$$

Total amount of deteriorated units

$$D=S-\int_0^{t_1}\frac{d}{(e-1)T}e^{\frac{t}{T}}dt$$

$$=S-\frac{d}{(e-1)T}\left[Te^{\frac{t}{T}}\right]_0^{t_1}$$

$$=S-\frac{d}{(e-1)}\left(e^{\frac{t_1}{T}}-1\right) \qquad \ldots(1.9)$$

Substituting the value of S from equation (1.3) in equation (1.6), we get

$$D=\frac{d}{(e-1)T}\left[t_1+\frac{t_1^2}{2T}+\frac{\alpha t_1^{\beta+1}}{\beta+1}+\frac{\alpha t_1^{\beta+2}}{T(\beta+2)}-Te^{\frac{t_1}{T}}+T\right]. \ldots(1.10)$$

Number of units in shortage

$$=\int_{t_1}^{T}I(t)dt$$

$$=\frac{d}{e-1}\left[2Te^{\frac{t_1}{T}}-t_1e^{\frac{t_1}{T}}-eT\right].$$

Therefore average shortage cost

$$=\frac{C_2 d}{T(e-1)}\left[2Te^{\frac{t_1}{T}}-t_1e^{\frac{t_1}{T}}-eT\right] \qquad \ldots(1.11)$$

Average holding cost

$$=\frac{1}{T}\int_0^{t_1} I(t)(h+\gamma t).dt$$

$$=\frac{1}{T}\int_0^{t_1} hI(t)dt+\frac{1}{T}\int_0^{t_1}\gamma tI(t)dt$$

$$=\frac{h}{T}\frac{d}{(e-1)T}\int_0^{t_1}\Bigg[t_1+\frac{t_1^2}{2T}+\frac{\alpha t_1^{\beta+1}}{\beta+1}+\frac{\alpha t_1^{\beta+2}}{T(\beta+2)}-t-\frac{t^2}{2T}$$

$$-\alpha t_1 t^{\beta}-\frac{\alpha t_1^2 t^{\beta}}{2T}+\frac{\alpha\beta t^{\beta+1}}{\beta+1}+\frac{\alpha\beta t^{\beta+2}}{2T(\beta+2)}\Bigg]dt$$

$$+\frac{\gamma}{T}\frac{d}{(e-1)T}\int_0^{t_1}\Bigg[t_1 t+\frac{t_1^2 t}{2T}+\frac{\alpha t_1^{\beta+1}.t}{\beta+1}+\frac{\alpha t_1^{\beta+2}}{T(\beta+2)}t-t^2$$

$$-\frac{t^3}{2T}-\alpha t_1 t^{\beta+1}-\frac{\alpha t_1^2 t^{\beta+1}}{2T}+\frac{\alpha\beta t^{\beta+2}}{\beta+1}+\frac{\alpha\beta t^{\beta+3}}{2T(\beta+2)}\Bigg]dt$$

$$=\frac{h}{T}\frac{d}{(e-1)T}\Bigg[t_1^2+\frac{t_1^3}{2T}+\frac{\alpha t_1^{\beta+2}}{\beta+1}+\frac{\alpha t_1^{\beta+3}}{T(\beta+2)}-\frac{t_1^2}{2}-\frac{t_1^3}{6T}$$

$$-\frac{\alpha t_1^{\beta+2}}{\beta+2}-\frac{\alpha t_1^{\beta+3}}{2T(\beta+1)}+\frac{\alpha\beta t_1^{\beta+2}}{(\beta+1)(\beta+2)}+\frac{\alpha\beta t_1^{\beta+3}}{2T(\beta+2)(\beta+3)}\Bigg]$$

$$+\frac{\gamma d}{T(e-1)T}\Bigg[\frac{t_1^3}{2}+\frac{t_1^4}{4T}+\frac{\alpha t_1^{\beta+3}}{2(\beta+1)}+\frac{\alpha t_1^{\beta+4}}{2T(\beta+2)}-\frac{t_1^3}{3}$$

$$-\frac{t_1^4}{8T}-\frac{\alpha t_1^{\beta+3}}{\beta+2}-\frac{\alpha t_1^{\beta+4}}{2T(\beta+2)}+\frac{\alpha\beta t_1^{\beta+3}}{(\beta+1)(\beta+3)}+\frac{\alpha\beta t_1^{\beta+4}}{2T(\beta+2)(\beta+4)}\Bigg]$$

$$=\frac{h}{T^2}\frac{d}{e-1}\Bigg[\frac{t_1^2}{2}+\frac{t_1^3}{3T}+\frac{\alpha\beta t_1^{\beta+3}}{2T(\beta+1)(\beta+2)}+\frac{\alpha\beta t_1^{\beta+2}}{(\beta+1)(\beta+2)}$$

$$+\frac{\alpha\beta t_1^{\beta+3}}{2T(\beta+2)(\beta+3)}\Bigg]+\frac{y}{T^2}\frac{d}{e-1}\Bigg[\frac{t_1^3}{6}+\frac{t_1^4}{8T}-\frac{\alpha\beta t_1^{\beta+3}}{2(\beta+1)(\beta+2)}$$

$$+\frac{\alpha\beta t_1^{\beta+3}}{(\beta+1)(\beta+3)}+\frac{\alpha\beta t_1^{\beta+4}}{2T(\beta+2)(\beta+4)}\Bigg]$$

$$=\frac{hd}{T^2(e-1)}\Bigg[\frac{t_1^2}{2}+\frac{t_1^3}{3T}+\frac{\alpha\beta t_1^{\beta+3}}{T(\beta+1)(\beta+3)}+\frac{\alpha\beta t_1^{\beta+2}}{(\beta+1)(\beta+2)}\Bigg]$$

$$+\frac{\gamma d}{T^2(e-1)}\Bigg[\frac{t_1^3}{6}+\frac{t_1^4}{8T}+\frac{\alpha\beta t_1^{\beta+3}}{2(\beta+2)(\beta+3)}+\frac{\alpha\beta t_1^{\beta+4}}{2T(\beta+2)(\beta+4)}\Bigg]. \quad \ldots(1.12)$$

Total average cost per time is given by

$$K(T_1)=\frac{CD}{T}+\text{Average Holding cost + Average Shortage cost}$$

$$=\frac{CD}{T}+\frac{hd}{T^2(e-1)}\left[\frac{t_1^2}{2}+\frac{t_1^3}{3T}+\frac{\alpha\beta t_1^{\beta+3}}{T(\beta+1)(\beta+3)}+\frac{\alpha\beta t_1^{\beta+2}}{(\beta+1)(\beta+2)}\right]$$
$$+\frac{\gamma d}{T^2(e-1)}\left[\frac{t_1^3}{6}+\frac{t_1^4}{8T}+\frac{\alpha\beta t_1^{\beta+3}}{2(\beta+1)(\beta+3)}+\frac{\alpha\beta t_1^{\beta+4}}{2T(\beta+1)(\beta+2)}\right]$$
$$+\frac{C_2 d}{T(e-1)}\left(2Te^{\frac{t_1}{T}}-t_1e^{\frac{t_1}{T}}-eT\right)$$

$$K(T_1)=\frac{cd}{T^2(e-1)}\left[t_1+\frac{t_1^2}{2T}+\frac{\alpha t_1^{\beta+1}}{\beta+1}+\frac{\alpha t_1^{\beta+2}}{T(\beta+2)}-Te^{\frac{t_1}{T}}+T\right]$$
$$+\frac{hd}{T^2(e-1)}\left[\frac{t_1^2}{2}+\frac{t_1^3}{3T}+\frac{\alpha\beta t_1^{\beta+3}}{T(\beta+1)(\beta+3)}+\frac{\alpha\beta t_1^{\beta+2}}{(\beta+1)(\beta+2)}\right]$$
$$+\frac{\gamma d}{T^2(e-1)}\left[\frac{t_1^3}{6}+\frac{t_1^4}{8T}+\frac{\alpha\beta t_1^{\beta+3}}{2(\beta+2)(\beta+3)}+\frac{\alpha\beta t_1^{\beta+4}}{2T(\beta+2)(\beta+4)}\right]$$
$$+\frac{c_2 d}{T(e-1)}\left(2Te^{\frac{t_1}{T}}-t_1e^{\frac{t_1}{T}}-eT\right) \qquad \text{...(1.13)}$$

The necessary conditions for minimum the total average costs $K(t_1,T)$ are

$$\frac{\partial K}{\partial T}=0 \quad \text{and} \quad \frac{\partial K}{\partial t_1}=0 \qquad \text{...(1.14)}$$

Now $\frac{\partial K}{\partial T}=0$, gives

$$-2C\left[t_1+\frac{3t_1^2}{4T}+\frac{\alpha t_1^{\beta+1}}{\beta+1}+\frac{3\alpha t_1^{\beta+2}}{2T(\beta+2)}-\frac{Te^{\frac{t_1}{T}}}{2}-\frac{t_1e^{\frac{t_1}{T}}}{2}+\frac{T}{2}\right]$$
$$-3h\left[\frac{t_1^2}{3}+\frac{t_1^3}{3T}+\frac{\alpha\beta t_1^{\beta+3}}{T(\beta+1)(\beta+3)}+\frac{2\alpha\beta t_1^{\beta+2}}{3(\beta+1)(\beta+2)}\right]$$
$$-3\gamma\left[\frac{2t_1^3}{9}+\frac{t_1^4}{8T}+\frac{\alpha\beta t_1^{\beta+3}}{2(\beta+2)(\beta+3)}+\frac{\alpha\beta t_1^{\beta+4}}{2T(\beta+2)(\beta+4)}\right]$$
$$+C_2t_1^2e^{\frac{t_1}{T}}=0. \qquad \text{...(1.15)}$$

and $\frac{\partial K}{\partial t_1}=0$, gives

$$C\left[1+\frac{t_1}{T}+\alpha t_1^{\beta}+\frac{\alpha t_1^{\beta+1}}{T}-e^{\frac{t_1}{T}}\right]$$

$$+h\left[t_1+\frac{t_1^2}{T}+\frac{\alpha\beta t_1^{\beta+1}}{\beta+1}+\frac{\alpha\beta t_1^{\beta+2}}{T(\beta+1)}\right]$$
$$+\gamma\left[\frac{t_1^2}{2}+\frac{t_1^3}{2T}+\frac{\alpha\beta t_1^{\beta+2}}{2(\beta+2)}+\frac{\alpha\beta t_1^{\beta+3}}{2T(\beta+2)}\right]$$
$$+C_2\left(Te^{\frac{t_1}{T}}-\frac{t_1}{T}e^{\frac{t_1}{T}}\right)=0. \qquad \ldots(1.16)$$

Equation (1.15) and (1.16) gives the optimum values of T and t_1 respectively. Provided

$$\frac{\partial^2 K}{\partial T^2}>0,\quad \frac{\partial^2 K}{\partial t_1^2}>0 \quad \text{and} \quad \left(\frac{\partial^2 K}{\partial T^2}\right)\left(\frac{\partial^2 K}{\partial t_1^2}\right)-\left(\frac{\partial^2 K}{\partial T\partial t_1}\right)^2>0 \ldots(1.17)$$

Case I: In case of finite planning horizon, the total average cost is given by

$$K(t_1)=\frac{Cd}{T^2(e-1)}\left[t_1+\frac{t_1^2}{2T}+\frac{\alpha t_1^{\beta+1}}{\beta+1}+\frac{\alpha t_1^{\beta+2}}{T(\beta+2)}-Te^{\frac{t_1}{T}}+T\right]$$
$$+\frac{hd}{T^2(e-1)}\left[\frac{t_1^2}{2}+\frac{t_1^3}{3T}+\frac{\alpha\beta t_1^{\beta+3}}{T(\beta+1)(\beta+3)}+\frac{\alpha\beta t_1^{\beta+2}}{T(\beta+1)(\beta+2)}\right]$$
$$+\frac{\gamma d}{T^2(e-1)}\left[\frac{t_1^3}{6}+\frac{t_1^4}{8T}+\frac{\alpha\beta t_1^{\beta+3}}{2(\beta+2)(\beta+3)}+\frac{\alpha\beta t_1^{\beta+4}}{2T(\beta+2)(\beta+4)}\right]$$
$$+\frac{C_2 d}{T(e-1)}\left[2Te^{\frac{t_1}{T}}-t_1e^{\frac{t_1}{T}}-eT\right] \qquad \ldots(1.18)$$

Sub case 1: When $\beta=1$
Sub case 2: When $\beta=2$
Sub case 3: When $\beta=3$

Case II: When T = 1, the total average cost is given by

$$K(t_1)=\frac{Cd}{e-1}\left[t_1+\frac{t_1^2}{2}+\frac{\alpha t_1^{\beta+1}}{\beta+1}+\frac{\alpha t_1^{\beta+2}}{\beta+2}-e^{t_1}+1\right]$$
$$+\frac{hd}{e-1}\left[\frac{t_1^2}{2}+\frac{t_1^3}{3}+\frac{\alpha\beta t_1^{\beta+3}}{(\beta+1)(\beta+3)}+\frac{\alpha\beta t_1^{\beta+2}}{(\beta+1)(\beta+2)}\right]$$
$$+\frac{\gamma d}{e-1}\left[\frac{t_1^3}{6}+\frac{t_1^4}{8}+\frac{\alpha\beta t_1^{\beta+3}}{2(\beta+2)(\beta+3)}+\frac{\alpha\beta t_1^{\beta+4}}{2(\beta+2)(\beta+4)}\right]$$
$$+\frac{C_2 d}{e-1}\left[2e^{t_1}-t_1e^{t_1}-e\right] \qquad \ldots(1.19)$$

Sub case I: When $\boldsymbol{\beta=1}$, then deterioration rate becomes constant in case of finite planning horizon

$$K(t_1)=\frac{cd}{T^2(e-1)}\left[t_1+\frac{t_1^2}{2T}+\frac{\alpha t_1^2}{2}+\frac{\alpha t_1^3}{3T}-Te^{\frac{t_1}{T}}+T\right]$$

$$+\frac{hd}{T^2(e-1)}\left[\frac{t_1^2}{2}+\frac{t_1^3}{3T}+\frac{\alpha t_1^4}{8T}+\frac{\alpha t_1^3}{6}\right]+\frac{\gamma d}{T^2(e-1)}$$
$$\left[\frac{t_1^3}{6}+\frac{t_1^4}{8T}+\frac{\alpha t_1^4}{24}+\frac{\alpha t_1^5}{30T}\right]+\frac{c_2 d}{T(e-1)}\left(2Te^{\frac{t_1}{T}}-t_1e^{\frac{t_1}{T}}-eT\right). \quad \ldots(1.20)$$

Therefore

$$\frac{dK(t_1)}{dt_1}=\frac{cd}{T^2(e-1)}\left[1+\frac{t_1}{T}+\alpha t_1+\frac{\alpha t_1^2}{T}-e^{\frac{t_1}{T}}\right]$$
$$+\frac{hd}{T^2(e-1)}\left[t_1+\frac{t_1^2}{T}+\frac{\alpha t_1^3}{2T}+\frac{\alpha t_1^2}{2}\right]+\frac{\gamma d}{T^2(e-1)}$$
$$\left[\frac{t_1^2}{2}+\frac{t_1^3}{2T}+\frac{\alpha t_1^3}{6}+\frac{\alpha t_1^4}{6T}\right]+\frac{c_2 d}{T^2(e-1)}\left(Te^{\frac{t_1}{T}}-t_1e^{\frac{t_1}{T}}\right)$$

For minimum total average cost $\frac{dK(t_1)}{dt_1}=0$

$$\Rightarrow \quad C\left(1+\frac{t_1}{T}+\alpha t_1+\frac{\alpha t_1^2}{T}-e^{\frac{t_1}{T}}\right)+h\left(t_1+\frac{t_1^2}{T}+\frac{\alpha t_1^3}{2T}+\frac{\alpha t_1^2}{2}\right)$$
$$+\gamma\left(\frac{t_1^2}{2}+\frac{t_1^3}{2T}+\frac{\alpha t_1^3}{6}+\frac{\alpha t_1^4}{6T}\right)+c_2\left(Te^{\frac{t_1}{T}}-t_1e^{\frac{t_1}{T}}\right)=0. \quad \ldots(1.21)$$

Sub case 2. When $\beta=2,$ deterioration rate will become a variable linear function of time, then total average cost becomes

$$K(t_1)=\frac{Cd}{T^2(e-1)}\left[t_1+\frac{t_1^2}{2T}+\frac{\alpha t_1^3}{3}+\frac{\alpha t_1^4}{4T}-Te^{\frac{t_1}{T}}+T\right]$$
$$+\frac{hd}{T^2(e-1)}\left[\frac{t_1^2}{2}+\frac{t_1^3}{3T}+\frac{2\alpha t_1^5}{15T}+\frac{\alpha t_1^4}{6}\right]+\frac{\gamma d}{T^2(e-1)}$$
$$\left[\frac{t_1^3}{6}+\frac{t_1^4}{8T}+\frac{\alpha t_1^5}{20}+\frac{\alpha t_1^6}{24T}\right]+\frac{c_2 d}{T^2(e-1)}\left(2T^2e^{\frac{t_1}{T}}-t_1Te^{\frac{t_1}{T}}-eT^2\right) \quad \ldots(1.22)$$

For minimum value of $K(t_1)$, $\frac{dK(t_1)}{dt_1}=0$

$$\Rightarrow \quad C\left(1+\frac{t_1}{T}+\alpha t_1^2+\frac{\alpha t_1^3}{T}-e^{\frac{t_1}{T}}\right)+h\left(t_1+\frac{t_1^2}{T}+\frac{2\alpha t_1^4}{3T}+\frac{2\alpha t_1^3}{3}\right)$$
$$+\gamma\left(\frac{t_1^2}{2}+\frac{t_1^3}{2T}+\frac{\alpha t_1^4}{4}+\frac{\alpha t_1^5}{4T}\right)+c_2\left(Te^{\frac{t_1}{T}}-t_1e^{\frac{t_1}{T}}\right)=0. \quad \ldots(1.23)$$

Sub case 3. When $\beta=3$, then deterioration rate will become a quadratic function of time. Then total average cost equation becomes

$$K(t_1)=\frac{Cd}{T^2(e-1)}\left[t_1+\frac{t_1^2}{2T}+\frac{\alpha t_1^4}{4}+\frac{\alpha t_1^5}{5T}-Te^{\frac{t_1}{T}}+T\right]$$
$$+\frac{hd}{T^2(e-1)}\left[\frac{t_1^2}{2}+\frac{t_1^3}{3T}+\frac{\alpha t_1^6}{8T}+\frac{3\alpha t_1^5}{20}\right]+\frac{\gamma d}{T^2(e-1)}$$
$$\left[\frac{t_1^3}{6}+\frac{t_1^4}{8T}+\frac{\alpha t_1^6}{20}+\frac{3\alpha t_1^7}{70T}\right]+\frac{c_2 d}{T(e-1)}\left[2Te^{\frac{t_1}{T}}-t_1e^{\frac{t_1}{T}}-eT\right]. \quad \text{...(1.24)}$$

For minimum value of $K(t_1)$, $\frac{dK(t_1)}{dt_1}=0$

$$\Rightarrow \quad C\left(1+\frac{t_1}{T}+\frac{\alpha t_1^3}{1}+\frac{\alpha t_1^4}{T}-e^{\frac{t_1}{T}}\right)+h\left(t_1+\frac{t_1^2}{T}+\frac{3\alpha t_1^5}{4T}+\frac{3\alpha t_1^4}{4}\right)$$
$$+\gamma\left(\frac{t_1^2}{2}+\frac{t_1^3}{2T}+\frac{3\alpha t_1^5}{10}+\frac{3\alpha t_1^6}{10T}\right)+c_2\left(Te^{\frac{t_1}{T}}-t_1e^{\frac{t_1}{T}}\right)=0. \quad \text{...(1.25)}$$

For minimum value of $K(t_1)$ $\frac{\partial K}{\partial t_1}=0$, which gives

$$C\left[1+t_1+\alpha t_1^{\beta}+\alpha t_1^{\beta+1}-e^{t_1}\right]+h\left[t_1+t_1^2+\frac{\alpha\beta t_1^{\beta+1}}{\beta+1}+\frac{\alpha\beta t_1^{\beta+1}}{\beta+1}\right]$$
$$+\gamma\left[\frac{t_1^2}{2}+\frac{t_1^3}{2}+\frac{\alpha\beta t_1^{\beta+2}}{2(\beta+2)}+\frac{\alpha\beta t_1^{\beta+3}}{2(\beta+2)}\right]$$
$$+C_2\left(Te^{t_1}-t_1e^{t_1}\right)=0. \quad \text{...(1.26)}$$

Case III. If γ = 0, then holding cost will becomes constant. Total averages cost becomes

$$K(t_1)=\frac{cd}{T^2(e-1)}\left[t_1+\frac{t_1^2}{2T}+\frac{\alpha t_1^{\beta+1}}{\beta+1}+\frac{\alpha t_1^{\beta+2}}{T(\beta+2)}-Te^{\frac{t_1}{T}}+T\right]$$
$$+\frac{hd}{T^2(e-1)}\left[\frac{t_1^2}{2}+\frac{t_1^3}{3T}+\frac{\alpha\beta t_1^{\beta+3}}{T(\beta+1)(\beta+3)}+\frac{\alpha\beta t_1^{\beta+2}}{(\beta+1)(\beta+2)}\right]$$
$$+\frac{c_2 d}{T(e-1)}\left(2Te^{\frac{t_1}{T}}-t_1e^{\frac{t_1}{T}}-eT\right). \quad \text{...(1.27)}$$

For minimum total averages cost, $\frac{dK(t_1)}{dt_1}=0$

$$\Rightarrow \quad C\left(1+\frac{t_1}{T}+\alpha t_1^{\beta}+\frac{\alpha t_1^{\beta+1}}{T}-e^{\frac{t_1}{T}}\right)+h\left(t_1+\frac{t_1^2}{T}+\frac{\alpha\beta t_1^{\beta+2}}{T(\beta+1)}+\frac{\alpha\beta t_1^{\beta+1}}{(\beta+1)}\right)$$
$$+c_2\left(Te^{\frac{t_1}{T}}-t_1e^{\frac{t_1}{T}}\right)=0. \quad \ldots(1.28)$$

Sub case 1. When $\beta=1$, then deterioration rate will become constant.

$$K(t_1)=\frac{Cd}{T^2(e-1)}\left[t_1+\frac{t_1^2}{2T}+\frac{\alpha t_1^2}{2}+\frac{\alpha t_1^3}{3T}-Te^{\frac{t_1}{T}}+T\right]$$
$$+\frac{hd}{T^2(e-1)}\left[\frac{t_1^2}{2}+\frac{t_1^3}{3T}+\frac{\alpha t_1^4}{8T}+\frac{\alpha t_1^3}{6}\right]$$
$$+\frac{C_2d}{T(e-1)}\left[2Te^{\frac{t_1}{T}}-t_1e^{\frac{t_1}{T}}-eT\right]. \quad \ldots(1.29)$$

For minimum value of $K(t_1)$, $\frac{dK(t_1)}{dt_1}=0$

$$\Rightarrow \quad C\left(1+\frac{t_1}{T}+\alpha t_1+\frac{\alpha t_1^2}{T}-e^{\frac{t_1}{T}}\right)+h\left(t_1+\frac{t_1^2}{T}+\frac{\alpha t_1^3}{2T}+\frac{\alpha t_1^2}{2}\right)$$
$$+C_2\left(Te^{\frac{t_1}{T}}-t_1e^{\frac{t_1}{T}}\right)=0. \quad \ldots(1.30)$$

Sub case 2. When $\beta=2$, then deterioration rate will become a variable linear function of time.

$$K(t_1)=\frac{Cd}{T^2(e-1)}\left[t_1+\frac{t_1^2}{2T}+\frac{\alpha t_1^3}{3}+\frac{\alpha t_1^4}{4T}-Te^{\frac{t_1}{T}}+T\right]$$
$$+\frac{hd}{T^2(e-1)}\left[\frac{t_1^2}{2}+\frac{t_1^3}{3T}+\frac{2\alpha t_1^5}{15T}+\frac{\alpha t_1^4}{6}\right]$$
$$+\frac{C_2d}{T(e-1)}\left[2T^2e^{\frac{t_1}{T}}-t_1Te^{\frac{t_1}{T}}-eT^2\right]. \quad \ldots(1.31)$$

For minimum value of $K(t_1)$, $\frac{dK(t_1)}{dt_1}=0$

$$\Rightarrow \quad C\left(1+\frac{t_1}{T}+\alpha t_1^2+\frac{\alpha t_1^3}{T}-e^{\frac{t_1}{T}}\right)+h\left(t_1+\frac{t_1^2}{T}+\frac{2\alpha t_1^4}{3T}+\frac{2\alpha t_1^3}{3}\right)$$
$$+C_2\left(Te^{\frac{t_1}{T}}-t_1e^{\frac{t_1}{T}}\right)=0. \quad \ldots(1.32)$$

Sub case 3. When $\beta=3$, then deterioration rate will become a quadratic function of time. Then total average cost

$$K(t_1)=\frac{Cd}{T^2(e-1)}\left[t_1+\frac{t_1^2}{2T}+\frac{\alpha t_1^4}{4}+\frac{\alpha t_1^5}{5T}-Te^{\frac{t_1}{T}}+T\right]$$
$$+\frac{hd}{T^2(e-1)}\left[\frac{t_1^2}{2}+\frac{t_1^3}{3T}+\frac{\alpha t_1^6}{8T}+\frac{3\alpha t_1^5}{20}\right]$$
$$+\frac{C_2 d}{T(e-1)}\left[2Te^{\frac{t_1}{T}}-t_1Te^{\frac{t_1}{T}}-eT\right]. \qquad \ldots(1.33)$$

For minimum total average cost, $\frac{dK(t_1)}{dt_1}=0$,

$$\Rightarrow \quad C\left(1+\frac{t_1}{T}+\alpha t_1^3+\frac{\alpha t_1^4}{T}-e^{\frac{t_1}{T}}\right)+h\left(t_1+\frac{t_1^2}{T}+\frac{3\alpha t_1^5}{4T}+\frac{3\alpha t_1^4}{4}\right)$$
$$+C_2\left(Te^{\frac{t_1}{T}}-t_1e^{\frac{t_1}{T}}\right)=0. \quad \ldots(1.34)$$

CONCLUSION

In this chapter, we have endeavored to develop a two-warehouse inventory system with a very realistic and practical deterioration rate. The effect of deterioration of physical goods in stock is very realistic feature of inventory control. In this model deterioration rate at any item is assumed to follow two parameter Weibull distribution function of time. This deterioration rate is suitable for items with and without life-period. The two warehouse inventory problem is an intriguing yet practical topic of decision science. The two-warehouse model can be applied to many practical situations, due to introduction of open market policy; the business competition becomes very high to occupy maximum possible market. As a result, the management of the departmental store is bounded to hire a separate warehouse on rental basis at a distance place for storing of excess items. Complete backlogged shortages are permitted in this study. Finally, the associated total cost minimization was illustrated by numerical exemplar and sensitivity analysis was also carried out by using MATHEMATICA–8.0 for the feasibility and applicability of the model. The results have also been interpreted graphically.

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